Neutrino-Astromony

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1.1 Neutrino flavor states

"Neutrino with a given flavor" identifies the neutral particle associated by weak interactions to the charged lepton with a given flavor, ie.e electron, muon or tau. EG ν_e in $e^- +^{37} Ar \rightarrow \nu_e +^{32} Cl$.

From observation: Flavor is conserved in the interaction point. Similar def. holds true for antineutrinos. Assumption (consistent with all known facts): Weak interactions are describe by a realtivistic qft. High energy: Standard electroweak model: (based $SU(3) \times SU(2) \times U(1)$). Assumption: Neutrino masses play a negligible role in interactions, i.e. ultra-relativistic limit $E_\nu \gg m_\nu$ holds.

1.2 Neutrino mixing

Assumption: Quantized neutrino fields with given flavor ν_α od not coincide with quantized field that have a given, defined mass, but rather coincide with linear combinations of fields ν_i that have mass m_i.

$$\nu_\alpha = \sum_{i=1}^{N} U_{\alpha i} \nu_i \text{ with } \alpha = e, \mu, \tau \; ; \; i = 1, 2, 3 \tag{1.1}$$

- ν_α do not have, in general, a definite mass.
- $U_{\alpha i}$ elements of a "leptonic mixing matrix"
- $U_{\alpha i}$ elements of a unitary 3x3 matrix (assumption!)

Consistent with:

1. Measured with Z^0 bosons
2. Neutrinos with given flavor have the same interactions - they are universal
3. Cosmological observations

Currently not observed:

1. More than 3 neutrinos, would violate unitarity
2. non-universal behaviour $\rightarrow$ no additional interactions

1.3 Neutrino transformations

Oscillating transformations $\nu_\alpha \rightarrow \nu_\beta (\alpha, \beta = e, \mu, \tau)$ in which one neutrino type transforms into another one with a different lepton flavor number: $L_\alpha \neq L_\beta$.

Necessary conditions:

1. Not all neutrinos have identical mass in particular not all neutrinos are massless
2. Lepton flavor numbers are not strictly conserved

Caveat: In SM neutrinos are assumed to be massless and L_α are conserved.

1.4 Neutrino oscillation phenomenology

Consider: n orthonormal eigenstates $|\nu_\alpha\rangle$:

$$\begin{aligned} \langle\nu_\alpha|\nu_\beta\rangle &= \delta_{\alpha,\beta} \\ \langle\nu_i|\nu_j\rangle &= \delta_{ij} \end{aligned}$$

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}\,|\nu_i\rangle$$

$$\begin{aligned} |\nu_i\rangle &= \sum_\alpha \left(U^\dagger\right)_{i\alpha}|\nu_\alpha\rangle \\ &= \sum_\alpha U^*_{\alpha i}\,|\nu_\alpha\rangle \end{aligned}$$

With unitarity:

$$\begin{aligned} U^\dagger U &= 1 \\ U^{-1} = U^\dagger &= \left(U^*\right)^{\mathrm{T}} \\ \text{i.e. } \sum_i U_{\alpha i}U^*_{\beta i} &= \delta_{\alpha\beta} \\ \sum_\alpha U_{\alpha i}U^*_{\alpha j} &= \delta_{ij} \\ |\overline{\nu_\alpha}\rangle = \sum_i U^*_{\alpha i}\,|\overline{\nu_i}\rangle &: \text{ antineutrinos} \end{aligned}$$

→ Different from $|\nu_\alpha\rangle$ if U_i is real
→ State defined in the ultrarelativistic limit are well-defined ($\lambda = -1$ for antineutrino) coinciding with chirality.

- Lowest-energy neutrinos detected today $\sim 200\,\mathrm{keV}$
- Conservative boundaries on neutrino mass: $\ll 2\,\mathrm{eV}$
  → Deviation from ultrarelativistic limit $\epsilon < 10^{-5}$

Breakdown?

- Near the endpoint of the β-spectrum: To conserve energy only ν_1 can be emitted with e^-
- Neutrinos produced in Big Bang now have momenta $p \sim kT \simeq 0.2\,\text{meV}$
  From $\sqrt{|\Delta m_{13}^2|} \simeq 50\,\text{meV}$ and $\sqrt{\Delta m_{12}^2} = 8.6\,\text{meV}$

1.5 Mixing Matrix

- Number of parameters of a unitary $n \times n$ matrix: n^2
- $2n$ neutrino states
  $\rightarrow 2n-1$ relative phases between states:
  $\rightarrow (n-1)^2$ independent parameters to describe phases

Example 1.1. 6 neutrinos, 5 relative phases, 4 parameter to fix 5 phases

Convention:

- $\frac{1}{2}n(n-1)$ "weak mixing angles" of an n-dimensional rotation matrix
- $\frac{1}{2}(n-1)(n-2)$ "CP-violating phases"

Remark. If neutrinos are majorana particles (i.e. their own antiparticles) 2 additional phases α_1 and α_2 occur. These cannot be observed in oscillation phenomena.

1.6 Mass eigenstates

Flavor eigenstates have no sharp, well-defined mass in general, i.e.

$$\langle \nu_\alpha | M | \nu_\beta \rangle \neq 0$$

But

$$\langle \nu_i | M | \nu_j \rangle = m_i \delta_{ij} \text{ and } m_i - m_j \neq 0\ \forall i, j : i \neq j$$

$|\nu_i\rangle$ is a stationary state with time dependency:

$$|\nu_i(x,t)\rangle = \left|\nu_i(0,0) e^{-ipx}\right\rangle$$
$$= |\nu_i(x,t)\rangle = \left|\nu_i(0,0) e^{-i(E\cdot t - \vec{p}\vec{x})}\right\rangle$$

with $E = \hbar\omega$ and $\vec{p} = \hbar\vec{k} = \hbar\frac{2\pi}{\lambda}\vec{e_k}$. Probability amplitudes propagate wave-like in space and time. Particle:

$$t = 0; x = 0 : \nu_1(0,0)$$

$$\text{at } x = L : |\nu_1(L,t)\rangle = \left|\nu_1(0,0)e^{-i(E_1\cdot t - p_1 L)}\right\rangle$$

Consider: 2 particles with mass m_1 and m_2 and the same energy E start to propagate in phase at $x = 0$ and $t = 0$ into the same direction. Phases at $x = L$:

$$|\nu_1(L,t)\rangle = \left|\nu_1(0,0)e^{-i(E\cdot t - p_1 L)}\right\rangle = \left|\nu_1(0,0)e^{-1\phi_1}\right\rangle$$
$$\nu_2(L,t) = \left|\nu_2(0,0)e^{-i(E\cdot t - p_2 L)}\right\rangle$$
$$= \left|\nu_2(0,0)e^{-i(Ecdott - p_1 L(p_1 - p_2)L)}\right\rangle$$
$$= \left|\nu_2(0,0)e^{-i\phi_1}e^{-i(p_1-p_2)L}\right\rangle$$

For $E \gg m$:

$$p_1 - p_2 = \sqrt{E^2 - m_1^2} - \sqrt{E^2 - m_2^2}$$
$$\simeq \left(E - \frac{m_1^2}{2E}\right) - \left(E - \frac{m_2^2}{2E}\right) = \frac{m_2^2 m_1^2}{2E} = \frac{\Delta m^2}{2E}$$

$$|\nu_2(L,t)\rangle = \left|\nu_2(0,0)e^{-ip_1}e^{i\frac{L\Delta m^2}{2E}}\right\rangle$$

$$\Delta = \frac{L}{2E}\Delta m^2$$

Time evolution of an eigenstate

$$|\nu_i(t)\rangle = e^{-iE_i t}|\nu_i\rangle$$

(Assumptions: $E_i \in \mathbb{R}$ and suppressing x-dependency)

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \simeq E + \frac{m_i^2}{2E}$$

for $p \gg m$ with $E \simeq p$ the neutrino energy. Pure flavor state ν_α at $t = 0$ evolves into $\nu(t)$:

$$|\nu(t)\rangle = \sum_{i=1}^{3} U_{\alpha i}|\nu_i\rangle e^{iE_i t} = \sum_{i.\beta} U_{\alpha i}U^*_{\beta i}e^{-iE_i t}\vec{\nu\beta}$$

Time-dependent transition amplitude for flavor transition $\nu_\alpha \to \nu_\beta$:

$$A(\alpha \to \beta; t) = \left\langle \nu_\beta \middle| \underbrace{\nu(t)}_{\nu_\alpha(0\to t)} \right\rangle = \sum_i U_{\alpha i}U^*_{\beta i}e^{-iE_i t} = \left(UDU^\dagger\right)_{\alpha\beta}$$

with $D_{ij} = \delta_{ij}e^{-iE_i t}$ a diagonal matrix.

$$A(\alpha \to \beta; t) = \sum_i U_{\alpha i}U^*_{\beta i}e^{-i\frac{m_i^2}{2}\cdot\frac{L}{E}} = A(\alpha \to \beta; L) \text{ with } L = c\cdot t$$

the distance between ν_α source and detector where ν_β is observed.

Remark. Phase-factor $e^{-iE\cdot t}$ cancels with e^{ipx} because of $E \simeq p$ and $x = t$ as neutrino propagate with close to the speed of light.

$$A(\overline{\alpha} \to \overline{\beta}, t) = \sum_i U_{\alpha i}^* U_{\beta i} e^{-iE_i t}$$